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Theories with Two Times

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ABSTRACT

General considerations on the unification of A-type and B-type supersymmetries in the context of interacting p-branes strongly suggest that the signature of space-time includes two timelike dimensions. This leads to the puzzle of how ordinary physics with a single timelike dimension emerges. In this letter we suggest that the two timelike dimensions could be real, and belong to two physical sectors of a single theory each containing its own timelike dimension. Effectively there is a single time evolution parameter. We substantiate this idea by constructing certain actions for interacting p-branes with signature $(n, 2)$ that have gauge symmetries and constraints appropriate for a physical interpretation with no ghosts. In combination with related ideas and general constraints in S-theory, we are led to a cosmological scenario in which, after a phase transition, the extra timelike dimension becomes part of the compactified universe residing inside microscopic matter. The internal space, whose geometry is expected to determine the flavor quantum numbers of low energy matter, thus acquires a Minkowski signature. The formalism meshes naturally with a new supersymmetry in the context of field theory that we suggested in an earlier paper. The structure of this supersymmetry gives rise to a new Kaluza-Klein type mechanism for determining the quantum numbers of low energy families, thus suggesting that the extra timelike dimension would be taken into account in understanding the Standard Model of particle physics.

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1 Hidden dimensions from generalized SUSY

Duality properties of string theory have shown that the basic theory behind string theory contains open and closed super p-branes interacting with each other. The fundamental form of the theory is unknown, however an overall property of the theory is the presence of A and B sectors related to each other by duality, such that in each sector a superalgebra with as many as 32 real supercharges and as many as 528 bosonic charges govern general properties of the interactions (this includes the heterotic sectors with 16 supercharges). The 32 supercharges correspond to the maximum number $N=8$ conserved Majorana supercharges in four flat dimensions. The 528 bosons correspond to all possible open and closed p-brane sources that can couple in all closed/open sectors. Some or all of these fermionic/bosonic charges may vanish in different sectors, including heterotic sectors. Thus, whatever the form of the theory, in its flat limit it must obey a generalized form of the superalgebra. The maximum amount of information is obtained when all 32 supercharges are active.

By examining various reclassification schemes of the A,B superalgebras it was recognized that quantum numbers associated with hidden dimensions may be attached to the $N=8$ labels in several ways [1]-[2]. In four dimensions one may distinguish 3 classification schemes of the internal $N=8$ labels that *are not contained in each other*. Namely $N=8$ corresponds to $8 \oplus 8^*$ of $SU(8)$, or to the spinors $8_+ \oplus 8_-$ of $SO(7,1)$, or to the spinors $(4,2) \oplus (4^*,2)$ of $SO(6) \times SO(2,1)$. Obviously these groups are not contained in each other. We refer to these as the duality basis, the A basis and the B basis respectively. Once the 32 spinors are classified, the 528 bosons obtain a unique classification since they appear in the products of the spinors. The transformations from one basis to another are related to duality transformations. Each classification includes an $SO(6)$ corresponding to 6 compactified dimensions already familiar from string theory. In the references above it was argued that the meaning of $SO(7,1)$ is 7 spacelike and 1 timelike internal dimensions, and that the meaning of $SO(6) \times SO(2,1)$ is 8 spacelike and 1 timelike internal dimensions. The $SU(8)$ classification is useful for studying certain duality properties of the theory but this basis obscures the other two classifications that give information about hidden dimensions in the theory (recall examples where electric-magnetic duality is obscured by Lorentz covariance and vice versa).

The three classifications are present in various dimensions. The content of the highest hidden dimensions is clearly displayed by rewriting the A,B type superalgebras directly in $SO(10,2)$ and $SO(9,1) \times SO(2,1)$ covariant forms respectively, suggesting that altogether there may be as many as 13 dimensions [2]. It was then noted that the A,B superalgebras correspond to two distinct projections from a bigger superalgebra that has 64 fermions and 2080 bosons ($=78+286+1716$) which are covariant under $SO(11,2)$. This suggested that the underlying unified theory of p-branes may be formulated in (11,2) dimensions. In recent work [3] it has been argued that the projection down to the A,B sectors that contain the two types $32_{A,B}$ supercharges may be implemented in a $SO(11,2)$ covariant form as a BPS condition. Furthermore, it was pointed out that the same 64 fermions and 2080 bosons ($=364+1716$) are actually covariant under $SO(11,3)$ and the A,B projections may be implemented with an $SO(11,3)$ covariant BPS condition. Therefore the fundamental supersymmetric theory may admit as many as 11 spacelike and 3 timelike dimensions, and this number of dimensions may be a limit due to supersymmetry.

If taken seriously this would suggest that at the fundamental level there are p-branes $X^M(\tau, \sigma_1, \dots, \sigma_p)$ where the target spacetime index M describes signature (11, 2) or (11, 3) in order to unify all sectors of the theory. For supersymmetry one should also take the 64 spinorial fermions $\theta_\alpha(\tau, \sigma_1, \dots, \sigma_p)$, with appropriate gauged kappa supersymmetries. The immediate issue that arises is how to construct a physical theory that contains such extra timelike dimensions? In particular how would one avoid the problems of two or more timelike dimensions, such as ghosts, causality, etc., that have been noted over many past decades?

Evidently there should be enough gauge symmetry in the construction of an action while preserving covariance. The gauge symmetry should lead to appropriate covariant constraints that permit a physical interpretation equivalent to a single time evolution parameter. This is the problem for which we suggest a solution. In the next section a covariant gauge mechanism is illustrated in an instructive toy model of interacting 0-branes, i.e. particles. The simple mechanism that solves the problem is generalized to strings and other p-branes in additional work elsewhere [4]. Our approach makes connections to other arguments and models about the fundamental theory that also involve (10,2) signature [5]-[15], and suggests the natural setting that leads to such models as part of a more complete system. The solution provides

the proper interpretation of our recent construction of field theoretic models with a new supersymmetry. Combining these observations with general properties of the superalgebra pointed out in the context of S-theory [2][16] (in particular as related to black holes) lead us to a cosmological scenario for the emergence of the standard universe with a single time coordinate from a theory of p-branes with more than one timelike dimensions. The extra timelike dimension(s) have consequences for the flavor quantum numbers in low energy physics.

2 Two particles

Consider the world lines of two particles (zero branes) $x_1^\mu(\tau)$, $x_2^\mu(\tau)$ with the following action

$$\begin{aligned} S &= S_1 + S_2 + S_{12} \\ S_n &= \frac{1}{2} \int_0^T d\tau \left(e_n \dot{x}_n^2 - \frac{m_n^2}{e_n} \right) \\ S_{12} &= \eta_{\mu\nu} \int_0^T d\tau \dot{x}_1^\mu A_1 \int_0^T d\tau \dot{x}_2^\nu A_2 \end{aligned} \tag{1}$$

where the familiar $S_1(x_1^\mu, e_1)$, $S_2(x_2^\mu, e_2)$ describe the free motion of each particle¹. S_{12} is an interaction involving the additional worldline fields $A_1(\tau)$, $A_2(\tau)$. This action is invariant under separate global spacetime translations of both x_1^μ and x_2^μ and *common* global rotations of all spacetime directions. The signature of the flat spacetime metric $\eta_{\mu\nu}$ *will not be a priori specified*, and it will be shown that the solution space *requires* a signature with at least two timelike dimensions. It will also be shown that the propagation of each particle can depend on only one linear combination of these timelike dimensions, while the other particle propagates in the orthogonal timelike dimension, thus each particle being consistent with propagation with a single time coordinate. This will be interpreted physically in a cosmological scenario.

¹If one substitutes the solutions for $e_{1,2}$ in the actions, then $S_{1,2}$ take the more familiar form $m_{1,2} \int d\tau \sqrt{-\dot{x}_{1,2}^2}$.

The action is invariant under two *independent* τ reparametrizations of the fields $[x_n^\mu(\tau), e_n(\tau), A_n(\tau)]$ for each $n = 1, 2$ which allow the two gauge choices $e_n = 1$ eventually. There are also two independent gauge invariances associated with the fields A_n given by

$$\begin{aligned}\delta x_1^\mu &= \lambda_2^\mu \Lambda_1(\tau), & \delta A_1 &= -\left(e_1 + \frac{\lambda_2^2 A_1}{\lambda_2 \cdot \dot{x}_1}\right) \partial_\tau \Lambda_1, \\ \delta x_2^\mu &= \lambda_1^\mu \Lambda_2(\tau), & \delta A_2 &= -\left(e_2 + \frac{\lambda_1^2 A_2}{\lambda_1 \cdot \dot{x}_2}\right) \partial_\tau \Lambda_2.\end{aligned}\tag{2}$$

where we have defined the dynamically determined constant vectors

$$\lambda_1^\mu = \int_0^T d\tau \dot{x}_1^\mu A_1, \quad \lambda_2^\mu = \int_0^T d\tau \dot{x}_2^\mu A_2.\tag{3}$$

Note that the fields labeled by $n = 1, 2$ are mixed by this gauge transformation since λ_2^μ appears in the transformations of particle #1, and vice-versa. It is due to these gauge invariances, which can remove one component from each $x_n^\mu(\tau)$, that effectively there is a single time coordinate. However, as we will see shortly, the dynamics will determine the two λ_n^μ to be orthogonal and timelike (or lightlike in the case of vanishing masses) thus requiring a signature $(n, 2)$ with two or more timelike dimensions.

Each particle moves in a dynamically defined background λ_n^μ provided by the other particle. For particle #1 moving in the background of particle #2 the effective action is

$$S_{1(2)} = \frac{1}{2} \int_0^T d\tau \left(e_1 \dot{x}_1^2 - \frac{m_1^2}{e_1} \right) + \lambda_{2\mu} \int_0^T d\tau \dot{x}_1^\mu A_1\tag{4}$$

The last term may be interpreted as an interaction of the particle with an “electromagnetic” potential of a specific form

$$A_\mu(\tau) = \lambda_{2\mu} A_1(\tau),\tag{5}$$

where $\lambda_{2\mu}$ plays the role of a polarization vector. The canonical momentum is conserved due to the equation of motion

$$p_1^\mu = e_1(\tau) \dot{x}_1^\mu(\tau) + \lambda_2^\mu A_1(\tau), \quad \partial_\tau p_1^\mu = 0.\tag{6}$$

The vector λ_1^μ is rewritten in terms of canonical variables

$$\lambda_1^\mu = \int_0^T d\tau \dot{x}_1^\mu A_1 = \int_0^T d\tau \hat{p}_1^\mu A_1 / e_1 \quad (7)$$

where

$$\hat{p}_1^\mu = p_1^\mu - \lambda_2^\mu A_1 \quad (8)$$

satisfies constraints and equations of motion that follow from the action

$$\lambda_2 \cdot \hat{p}_1 = 0, \quad \hat{p}_1^2 + m_1^2 = 0, \quad \partial_\tau \hat{p}_1^\mu = 0. \quad (9)$$

The first constraint shows (without using the equation of motion or special gauges) that λ_1^μ is orthogonal to λ_2^μ

$$\lambda_1 \cdot \lambda_2 = 0. \quad (10)$$

The equation $\partial_\tau \hat{p}_1^\mu = 0$ follows from $\partial_\tau p_1^\mu = 0$ and the first constraint, because $A_1 = p \cdot \lambda_2 / \lambda_2^2$ is a constant provided the denominator does not vanish. If $\lambda_2^2 = 0$ then $A_1(\tau)$ is undetermined, but it can be chosen to be a constant by using the gauge freedom $\delta x_1^\mu = \lambda_2^\mu \Lambda_1(\tau)$, $\delta A_1 = -e_1 \partial_\tau \Lambda_1$. Either way $\partial_\tau \hat{p}_1^\mu = 0$ is satisfied. Since \hat{p}_1^μ is conserved then λ_1^μ is proportional to it

$$\lambda_1^\mu = \hat{p}_1^\mu \int_0^T d\tau A_1 / e_1 \quad (11)$$

The remaining constraint (mass shell condition) requires that λ_1^μ is timelike for $m_1 \neq 0$ or lightlike for $m_1 = 0$. Similar arguments hold for λ_2^μ . Choosing the gauges $e_n = 1$, the overall physical solution is given by

$$x_1^\mu(\tau) = \hat{p}_1^\mu \tau + q_1^\mu, \quad x_2^\mu(\tau) = \hat{p}_2^\mu \tau + q_2^\mu \quad (12)$$

$$\hat{p}_n^2 = -m_n^2, \quad \hat{p}_1 \cdot \hat{p}_2 = 0, \quad \lambda_n^\mu = \hat{p}_n^\mu A_n T, \quad (13)$$

where the A_n are constants.

According to the solution space the particles move freely except for the constraint that the momenta $\hat{p}_1^\mu, \hat{p}_2^\mu$ must be orthogonal. Furthermore, in the massive case, since both vectors are timelike the constraints cannot be satisfied with only one timelike dimension. One must have at least two timelike dimensions, therefore we take the signature for the spacetime index μ as $(n, 2)$. Yet, effectively each particle moves in a subspace of signature $(n, 1)$.

For example in the rest frame of particle #2 one can write the momenta in the form

$$\hat{p}_2^\mu = (m_2, 0; \mathbf{0}), \quad \hat{p}_1^\mu = \left(0, \sqrt{m_1^2 + \mathbf{p}_1^2}; \mathbf{p}_1\right) \quad (14)$$

showing that the energy-momentum relations are as usual. In addition, note that there is in fact only one proptime parameter τ to describe the time evolution (because it is possible to choose both $e_n = 1$).

If one of the particles is massless, say $m_2 = 0$, then $\lambda_2^\mu \sim \hat{p}_2^\mu$ is lightlike. As mentioned above, one may use the gauge freedom to fix A_1 . Thus, one may choose A_1 to be just the required constant so that $\hat{p}_1^\mu = p_1^\mu - \lambda_2^\mu A_1$ has no components along \hat{p}_2^μ . Therefore, the solution space is again as above, but now \hat{p}_1^μ has two less components, one timelike and one spacelike, since it cannot point along the lightlike vector \hat{p}_2^μ . So it propagates in a space of signature $(n-1, 1)$. For example, in a special frame one may write

$$\hat{p}_2^\mu = (|p_2|, 0; p_2, \vec{0}), \quad \hat{p}_1^\mu = \left(0, \sqrt{m_1^2 + \vec{p}_1^2}; 0, \vec{p}_1\right). \quad (15)$$

We emphasize that one may change

$$\hat{p}_1^\mu \rightarrow \hat{p}_1^\mu + \alpha \hat{p}_2^\mu \quad (16)$$

for any constant α , but this is a gauge freedom. By choosing $\alpha = 0$ we keep the interpretation that each particle uses only one timelike coordinate.

If both particles are massless $m_1, m_2 = 0$, the arguments above may be repeated to find out that the A_n may be chosen as constants just as to insure that \hat{p}_n^μ are time independent, lightlike and not parallel to each other (the last possibility is a gauge choice). In a special frame they may be written as in eq.(15) in the limit $m_1 = 0$.

The lightlike case for \hat{p}_2^μ is reminiscent of a lightlike vector introduced in the context of several attempts for constructing theories with signature $(n, 2)$. These include heterotic (p,q) strings involving left/right movers with N=2 supersymmetry [8][10], F-theory [9], (10, 2) super Yang-Mills [12], (10, 2) version of a matrix model for M-theory [13]. We suggest that a natural interpretation of the lightlike vector in these theories would be obtained by considering the type of system discussed in this paper. Namely consider the presence of a 0-brane or a p-brane #2 which provides a background to the strings considered in these models. Then in the sector of lightlike \hat{p}_2^μ the

models would be recovered, but there would be now the additional system #2 to be taken into account including all the allowed values and directions of the vector \hat{p}_2^μ . This will remove the rigidity of the lightlike vector and establish full $\text{SO}(n, 2)$ covariance for the full model. Such generalized models will be discussed in another paper [4].

The quantum theory of the two particle system can be written covariantly with $\text{SO}(n, 2)$ symmetry. One may start with naive quantization rules

$$[x_m^\mu, \hat{p}_n^\nu] = i\eta^{\mu\nu}\delta_{mn}, \quad [x_m^\mu, x_n^\nu] = [\hat{p}_m^\mu, \hat{p}_n^\nu] = 0 \quad (17)$$

and impose the constraints on the states. Since there are two particles, the wavefunction in position space depends on both coordinates with $(n, 2)$ signature, and satisfies a Klein-Gordon type equation corresponding to the constraints

$$(\partial_n^2 - m_n^2) \Phi(x_1^\mu, x_2^\mu) = 0, \quad \partial_1 \cdot \partial_2 \Phi(x_1^\mu, x_2^\mu) = 0. \quad (18)$$

The general solution is the general superposition of a product of plane waves with momenta that are on shell and orthogonal to each other

$$\begin{aligned} \Phi(x_1^\mu, x_2^\mu) = & \int d^{n+2}k_1 d^{n+2}k_2 \delta(k_1^2 + m_1^2) \delta(k_2^2 + m_2^2) \delta(k_1 \cdot k_2) \\ & [a_+(k_1, k_2) e^{i(k_1 \cdot x_1 + k_2 \cdot x_2)} + a_-(k_1, k_2) e^{i(k_1 \cdot x_1 - k_2 \cdot x_2)} + c.c.]. \end{aligned} \quad (19)$$

The delta functions impose the constraints. The coefficient $a_+(k_1, k_2)$ includes a theta function $\theta(k_1^0 k_2^{0'} - k_1^{0'} k_2^0)$ while a_- contains a theta function with the opposite argument. The sign of the argument of the theta function cannot be changed with $\text{SO}(n, 2)$ transformations, therefore including the theta functions is the analog of imposing the positive energy condition. Hence the coefficients a_\pm have the interpretation of probability amplitude for (particle #1, particle #2) and (particle #1, antiparticle #2) respectively. For their complex conjugates particle is interchanged with antiparticle. This interpretation of the coefficients is consistent with $\text{SO}(n, 2)$ covariance.

If one or both particles are massless additional conditions are needed to implement the gauge freedom of eq.(16) to insure that the timelike coordinates of the particles do not overlap. The gauge freedom (for the case $m_2 = 0$) corresponds to translating $k_1^\mu \rightarrow k_1^\mu + \alpha k_2^\mu$ and integrating over α . This produces an additional delta function $\delta(k_2 \cdot x_1)$ to be inserted in the

integral above. Similarly, if m_1 is also zero there would be another constraint implemented by $\delta(k_1 \cdot x_2)$. Of course the constrained momenta point in all possible directions in the spacetime of $(n, 2)$ signature to be able to recover all possible quantum states and full covariance under $\text{SO}(n, 2)$.

There are no ghosts in the spectrum of the first quantized theory. A field theory that gives precisely only such solutions can be written down as in our previous paper on a “new supersymmetry” [17]. The field theory that we proposed has higher derivative terms, but they are arranged in such a way that ghost problems are avoided as demonstrated there. Also, in [17] the field theory corresponds to a compactified version of the two particle problem, but this has been generalized to the general case in [18] with similar results.

3 New Supersymmetry

The supersymmetric version of the two particle field theory described above may be constructed along the lines suggested in our recent paper [17]. For simplicity we will consider $N = 1$ in $(n, 2)$ dimensions. Let the supercharge be denoted by Q_α . We had shown that for a superalgebra of the form

$$\{Q_\alpha, Q_\beta\} = \gamma_{\alpha\beta}^{MN} Z_{MN}, \quad Z^{MN} = \hat{p}_1^M \hat{p}_2^N - \hat{p}_1^N \hat{p}_2^M, \quad (20)$$

one can construct representations and Lagrangians in a field theory with bilocal fields $\Phi(x_1^\mu, x_2^\mu)$. Actually we had considered a compactified version of this form (see also below eq.(23)) but the approach is generalizable to the present case [18]. In our previous paper a two particle interpretation was not given but the formalism that was used was equivalent to it. With our current understanding the two particle interpretation of this superalgebra is quite natural.

If we use the constraints on the momenta derived in the previous section this superalgebra reduces to the standard one in a special frame of each particle. For example consider the case of massive particle #2. In its rest frame its momentum points along the $0'$ direction and the superalgebra reduces to

$$\{Q_\alpha, Q_\beta\} = 2m_2 \gamma_{\alpha\beta}^{0'\mu} Z_{0'\mu} = 2m_2 \left(\gamma^{0'} \gamma^\mu \right)_{\alpha\beta} p_{1\mu} \quad (21)$$

This is recognized as the one particle superalgebra for a spacetime with signature $(n, 1)$. If n is even, and if the spinor index α corresponds to a Weyl

spinor for $SO(n, 2)$, then Q_α is an irreducible spinor in $(n, 1)$ dimensions. If n is odd then Q_α splits into two spinors of opposite chirality in $(n, 1)$ dimensions.

For a massless particle #2 in its special frame the $\gamma^{0'}$ would be replaced by the lightcone combination $\gamma^{0'+1'}$ and the superalgebra would become the one particle superalgebra in a spacetime with signature $(n-1, 1)$. Half of the original supergenerators Q_α vanish because they satisfy a BPS condition. The other half is the spinor for $SO(n-1, 1)$ that forms the $N=1$ superalgebra in the lower dimension that describes the motion of particle #1 (reduced by one spacelike and one timelike dimensions).

For fields $\Phi(x_1^\mu, x_2^\mu)$ that describe all possible states which are not necessarily in special frames the superalgebra (20) becomes

$$\{Q_\alpha, Q_\beta\} = -2\gamma_{\alpha\beta}^{MN} \partial_{1M} \partial_{2N} \quad (22)$$

This is the form for which representations can be found as discussed in a previous paper [17] and a future one [18]. Such representations include bilocal bosonic and fermionic fields and generalize the representations of standard supersymmetry.

The special forms of the superalgebra (20,22) were first considered in the context of S-theory [2]. Mass shell conditions corresponding to BPS conditions which restricted \hat{p}_n^M were considered, including conditions such as $\hat{p}_1 \cdot \hat{p}_2 = 0$ as in the present paper. The form of Z^{MN} was an ansatz motivated by certain applications of S-theory, but at the time it was not known why such an algebra would arise. It was simply noted that it would be the minimal algebra required in a $(10, 2)$ version of supergravity. A compactified version, including central extensions, was also applied to the problem of labeling black hole states and computing the black hole entropy [16]. The two particle constrained problem introduced in the current paper provides a natural setting for this form of new supersymmetry and all of its previous applications. As insisted in S-theory and in its applications, the two momenta must be allowed to take all possible values in the spacetime of signature $(n, 2)$ in order to recover the full $SO(n, 2)$ covariance of the underlying theory. The two particle interpretation demands this naturally.

It is interesting to point out that it was later noted that the same form for the superalgebra appears in $(10, 2)$ versions of super Yang-Mills theory [12], $(2, 1)$ superstrings [11] and matrix models for M-theory [13]. However, since

in those applications \hat{p}_2^M was taken as a rigid lightlike vector, the $SO(10, 2)$ covariance was broken. The $SO(10, 2)$ covariance would be valid in a larger theory in which the lightlike vector is allowed to point in all possible directions [19][16]. This generalization is now justified by suggesting that those models can be reconsidered and interpreted naturally in a setting analogous to the two particle problem discussed in this paper and in [4].

4 Cosmological scenario and low energy physics

If at some deep level there is an extended universe that functions with two timelike dimensions, how does it evolve to our present universe in which there is only one time? It is common to assume that there was a phase transition during the early universe era, such as the Big Bang, which triggered the expansion of some of the dimensions while the others remained compact and small. In conjunction with this phase transition, a plausible scenario is that one of the timelike dimensions goes along with the expanding universe and the other goes along with the compactified one as described below. For instance, in the two particle model (or its generalization to p-branes) p_1^μ would be in four dimensions while p_2^m would be in the compactified dimensions including the extra timelike dimension. Amusingly, such a picture seems to be consistent with several other observations and therefore deserves some study, even though the physical mechanism that triggers the Big Bang remains obscure.

Let us first connect it to some observations about black holes, since sometimes it is useful to think of black hole singularities as the opposite of Big Bang type white hole singularities. In that case one may expect that the extra compactified timelike and spacelike dimensions reside also inside black holes. In fact, there is already a clue in favor of this picture in agreement with the scenario we propose, which indicates that black holes do have information about the hidden time and space coordinates. During the past year new advances were made in computing the entropy, for certain stringy charged black holes, in terms of D-branes [20]. The result shows that the black hole entropy is written in terms of the central extensions of the superalgebra [21] which describe the wrappings of p-branes in internal spacelike dimensions. By using the reclassification of the supercharges mentioned in the first section, it was also shown that the expression for the black hole

entropy corresponds to evaluating it in the rest frame of an internal Lorentz space $SO(c+1, 1)$ where $c+1$ is the number of compactified string coordinates c plus the 11th dimension, while the timelike direction corresponds to the 12th dimension. The same black hole entropy was then written in the general $SO(c+1, 1)$ Lorentz frame by introducing the momentum vector p_2^m (which appears in the superalgebra (20)) taken along *only the compactified dimensions* $(c+1, 1)$ [16]. The entropy is then an invariant of $SO(c+1, 1)$, as it should be, since counting states in representations of the superalgebra could not depend on the basis used to classify them. However, in the general Lorentz basis, the entropy is expressed in terms of the momentum p_2^m and central extensions that are classified by $SO(c+1, 1)$. Similarly, the mass of the black hole is expressed through the same quantities. Thus, such p-brane charged black holes “know” about the existence of the $(c+1, 1)$ dimensions as exhibited in the expressions for the entropy and mass. This is in agreement with the cosmological view expressed in this paper.

Encouraged by these observations, we propose the following plausible scenario: Before the Big Bang phase transition the fundamental theory could be a theory of interacting open and closed super p-branes propagating in a spacetime of signature $(11, 2)$ or $(11, 3)$. The form of the theory is unknown but it is assumed to obey the general algebra of S-theory. The interaction of the p-branes is taken such that there are constraints on their relative motion that effectively forces them to propagate with a single timelike coordinate (as in the two particle example discussed in section 2, and generalized in another paper to strings [4]). The constrained system still allows the p-branes to propagate in various linear combinations of the two timelike dimensions as illustrated by the solution $\Phi(x_1, x_2)$ in the two particle example. However, when the Big Bang phase transition takes place (for reasons that remain obscure at the present) some of the p-branes are associated with one timelike coordinate which parametrizes the expanding portion of the universe while others remain in the compactified space that includes the other timelike coordinate. For low energy physics it is that sector of solution space which becomes relevant. The information about the extra timelike coordinate would not be lost to observers or matter in the expanding universe because low energy physics would need to take into account the geometry of the internal space which would have Minkowski signature.

If one would take this scenario seriously then the superalgebra that would apply in low energy physics would naturally be along the lines of the “new

supersymmetry” discussed in our previous paper [17]. For example in four dimensions

$$\{Q_{\alpha a}, Q_{\dot{\beta} b}\} = 2\sigma_{\alpha\dot{\beta}}^{\mu}\gamma_{ab}^m p_{1\mu} p_{2m} + \text{extensions}, \quad (23)$$

where $p_{1\mu}$ is a vector in $\text{SO}(3, 1)$ corresponding to ordinary momentum in 4D and p_{2m} is a vector in $\text{SO}(c + 1, 1)$ corresponding to the momentum of the p-branes in the internal dimensions. The fermionic and bosonic fields that form representations of this superalgebra depend both on the 4D and internal coordinates $\Phi(x^{\mu}, y^m)$ as discussed in our previous work. A Kaluza-Klein type of expansion in a complete set of states of the internal geometry identifies the low energy massless families. It was shown in [17] that this formalism can provide a new source of family generation (and unification) that is different than the standard Kaluza-Klein mechanism. A simple example of how massless families could emerge while being unified in a new supermultiplet was given in [17]. The essential difference is that $\gamma_{ab}^m p_{2m}$ appears in the form of a product with $\sigma_{\alpha\dot{\beta}}^{\mu} p_{1\mu}$ in the case of the new supersymmetry, whereas it appears in the form of a sum (and only spacelike components in p_2) in the case of standard supersymmetry. The nature of the internal geometry would need to follow from more detailed theories, but as is already clear from the simple example, one would not be limited to only Calabi-Yau spaces in spacelike dimensions since the internal space would have Minkowski signature in the new scenario.

We do not claim to have resolved all issues related to two timelike dimensions in this paper, but we think we have illustrated how to surmount some of the major and obvious obstacles. Besides low energy physics one expects that the extra timelike dimension(s) would have some other consequences, for example in the early universe. Our approach needs to be applied and tested in cosmology. Since the mechanism we have suggested is consistent with and has clear connections to several lines of thought to the unknown theory behind string theory, it would be interesting to explore its consequences and its consistency in more detail by reconsidering those theories.

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